# HYDRODYNAMIC PERMEABILITY OF A FIBROUS MEMBRANE CONSISTING OF CYLINDRICAL CELLS WITH POROUS LAYER UNDER FLOW OF MICROPOLAR FLUID PARALLEL TO THE AXIS OF CYLINDERS

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## Introduction

Flow through random assemblage of particles have always been a topic of interest to researchers due to its vast applications in various real life problems such as flow through sand beds, petroleum reservoirs, membrane filtration processes etc. For modeling of flow through a porous medium, Darcy or Brinkman formulations are used, depending upon the media. Cell model technique is an approach to analyze such problems by taking one particle in the swarm to be confined in a hypothetical cell and applying appropriate boundary condition on the cell surface to include the effect of neighboring particles on the particle concerned. In this way, the problem of analyzing flow past each particle is reduced to flow past a single particle.

Usually the investigations by cell model technique were done for flow of Newtonian fluid. However, in many real life problems the fluid may be non-Newtonian in nature such as blood flow through arterial wall where animal blood shows characteristic of micropolar fluid. Apart from this, liquid crystals made up of dumbbell molecules, polymeric fluids with additives also behave like micropolar fluids. Eringen used conservation of micro inertia moments and balancing of first stress moments to develop theory of microfluids, which possess local inertia. This theory helped researchers to use inertial spin, body and stress moments, micro stress averages in deriving equations to get complete flow information in specific cases such as polymeric fluids with additives etc. Although micro-fluids take into consideration the micro-rotation and spin inertia but the theory become very complicated for a general class of micro fluids due to the complexities involved in formulations. A micropolar fluid is a subclass of microfluids, which supports couple stress, and body couples only, which makes its formulations relatively simple for researchers. In micropolar fluids, rotation of a fluid point in volume element about centroid is taken into account other than its rigid motion by means of microrotation vector.

Up to the knowledge of the authors the cell with the combined solid-porous core, flowed by a micropolar liquid has not been considered yet either for cylindrical or for spherical geometry. In the review [1] there was offered a general statement of the problem and various types of boundary conditions were discussed. This paper starts the set of three works devoted to the application of cell model with combined solid-porous core in micropolar flow. It considers the flow parallel to the symmetry axis of cylindrical cell. Two different boundary conditions on the surface of the cell will be considered. For both formulations, the expression for hydrodynamic permeability of a swarm has been obtained and effect of various parameters on hydrodynamic permeability is discussed.

## Theory

A cylindrical cell, shown in Fig.1, consists of a solid impermeable core of radius *a*, coaxial porous layer $a < r < b$ and outer region $b < r < c$ of free micropolar liquid flow. The squared relative thickness of porous layer in a cell $b^2 / c^2$ can be chosen to be equal to the volume fraction of particles in the dispersed system (membrane), which is connected to its porosity. The flow is directed along the symmetry axis of the cell and driven by the pressure gradient $-\nabla p$, which is equal to $-\partial p / \partial z$ in the cylindrical coordinate system $(r, \theta, z)$. The micropolar flow is governed by the system of three equations after Eringen: the continuity equation, the momentum equation and the moment of momentum equation

$$\nabla \cdot \mathbf{v} = 0,$$
$$\rho\dot{\mathbf{v}} = \rho\mathbf{F} - \nabla p + (\mu+\kappa)\Delta\mathbf{v} + 2\kappa\nabla\times\boldsymbol{\omega}, \quad (1)$$
$$\rho\hat{J}\dot{\boldsymbol{\omega}} = \rho\mathbf{L} + (\alpha+\delta-\varsigma)\nabla\nabla\cdot\boldsymbol{\omega} + (\delta+\varsigma)\Delta\boldsymbol{\omega} + 2\kappa\nabla\times\mathbf{v} - 4\kappa\boldsymbol{\omega},$$

where $\mathbf{v}, \boldsymbol{\omega}$ are linear and angular velocity vectors correspondingly, $\rho$ is the liquid density, $\hat{J}$ the moment of inertia tensor, $\mathbf{F}, \mathbf{L}$ are the densities of external forces and couples, $\mu, \kappa, \alpha, \delta, \varsigma$ are the viscosity coefficients of the micropolar medium.

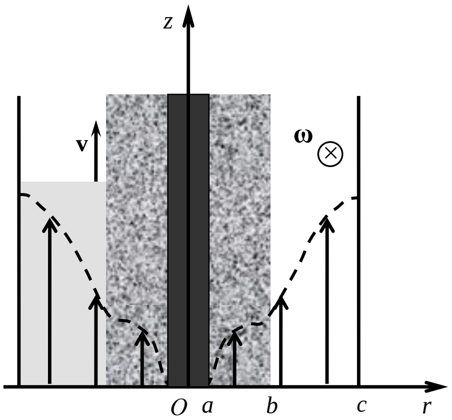



*Figure 1. Geometry of the flow through a cylindrical cell*

It's worth mentioning that system (1) slightly differs by the notation of coefficients from the original Eringen's formulation, which is used in the majority of papers. Namely, coefficients $\mu$ and $\kappa$ are introduced in such a manner that at $\kappa = 0$ not only the first and the second equations of system (1) are reduced to the Navier-Stokes equations of classical hydrodynamics, but also the viscosity coefficient $\mu$ is equal to the dynamic viscosity of Newtonian liquid. Detailed consideration of the governing equations formulation and viscosity coefficients definition is given in [1].

Brinkman-type equations will be used for the modeling of the flow in a porous region. According to the original Brinkman's idea developed for the stationary filtration flows of non-polar liquids [2], the applied pressure gradient is balanced by the viscous terms and the resistance generated by the porous matrix. The former enters the equation with the coefficient called effective viscosity $\mu'$, which is not necessarily equal to the dynamic viscosity $\mu$. The porous matrix was approximated as a swarm of spherical particles with the drag described by the Stokes formula, which entered the equation with the coefficient *k* called specific permeability and in fact represents the known Darcy term. Later, Ochoa-Tapia and Whitaker rigorously derived the Brinkman equation by the averaging method over the representative volume. It was shown also that the effective viscosity is equal to $\mu/\varepsilon$, where $\varepsilon$ is the local porosity of the medium (in our case – porous layer surrounding solid core). Adopting the original Brinkman's idea for the micropolar flow in a porous medium one should write the momentum equation with viscous terms corrected by the coefficient $\varepsilon^{-1}$ and add Darcy-like resistance term calculated for the micropolar flow. Many authors considered the problem of the micropolar flow over the sphere of radius *R* with various boundary conditions. First found was the drag force with no-slip and no-spin conditions. It has the most concise expression, and by the notations of the present paper takes the form $F_0 = 6\pi UR\mu\left(1+\frac{\kappa}{\mu}\bigg/\left(1+2R\sqrt{\frac{\kappa(\mu+\kappa)}{\mu(\delta+\varsigma)}}\right)\right)$. For a sphere of infinitesimal radius this expression can be reduced to $F = 6\pi UR(\mu+\kappa)$. So, the Darcy-like term for micropolar fluid differs the classical Darcy term by the coefficient $\mu+\kappa$. The moment of momentum equation should be remained unchanged, since couple on the sphere is zero. Rigorous mathematical derivation of the equations governing the filtration of micropolar flow is fulfilled in the paper of Kamel et al. [2]. The standard averaging technique over the representative volume is used and the no-slip, no-spin conditions are applied on solid boundaries. The resultant system of equations for the averaged values of stationary slow flow can be written in the form

$$\nabla \cdot \mathbf{v} = 0,$$

$$\nabla p = \frac{\mu+\kappa}{\varepsilon}\Delta\mathbf{v} + 2\frac{\kappa}{\varepsilon}\nabla\times\boldsymbol{\omega} - \frac{\mu+\kappa}{k}\mathbf{v}, \qquad (2)$$

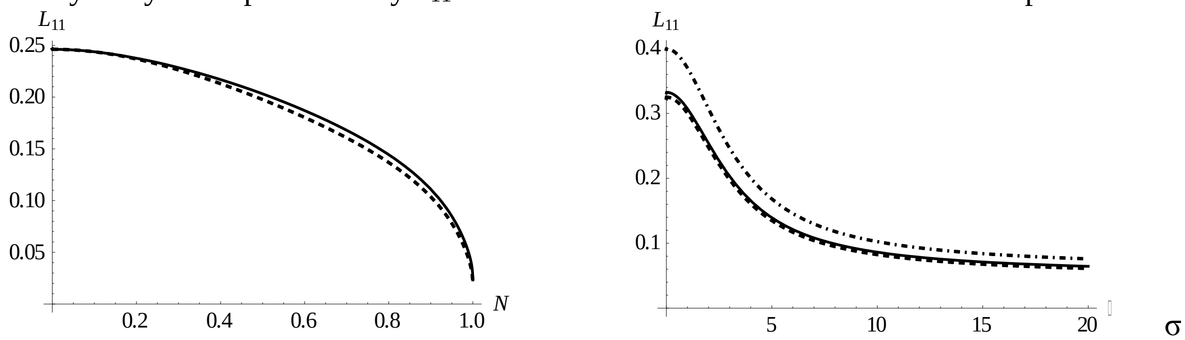
$$0 = (\alpha+\delta-\varsigma)\nabla < \nabla\cdot\boldsymbol{\omega} > + (\delta+\varsigma)\Delta\boldsymbol{\omega} + 2\kappa\nabla\times\mathbf{v} - 4\kappa\boldsymbol{\omega},$$

where the spin divergence averaged over the representative volume $V$ is $<\nabla\cdot\boldsymbol{\omega}> = \frac{1}{V}\int_V \nabla\cdot\boldsymbol{\omega}\, dV$.

For the cylindrical geometry and some other cases important for practice, the condition $\nabla\cdot\boldsymbol{\omega} = 0$ is fulfilled, and this term vanishes. Thus, the derived equations are totally coincided with those which would be obtained by simple mechanical consideration suggested by Brinkman.

## Results and discussion

We applied governing equations (1) and (2) and appropriate boundary conditions for the dispersion system (membrane) one cell of which is shown in Fig.1 in order to solve the BVPs and find hydrodynamic permeability $L_{11}$ of the membrane. The results obtained are depictured in Fig.2.




*(a)* *(b)*

*Figure 2. Variation of hydrodynamic permeability with coupling parameter N (a) and with parameter* $\sigma = b/\sqrt{k}$ *(b) under no couple stress condition (solid line), no spin condition (dashed line) on the outer surface of the cell and for the Newtonian liquid (dot-dashed line)*

The most interesting is the effect of micropolar properties of the liquid on the flow. It is determined by parameters $N = \sqrt{\kappa/(\mu+\kappa)}$, $L = \sqrt{(\delta+\varsigma)/\mu}\big/(2b)$, $\phi = (\delta-\varsigma)/(\delta+\varsigma)$ with following constraints: $N \in [0;\ 1)$; $L \in [0;\ \infty)$; $\phi \in [-1;\ 1]$. Fig.2a shows the dependency of $L_{11}$ on $N$ for both problem formulations, i.e. using *no couple stress condition* and *no spin condition*. There is observed a monotonous and significant decay of hydrodynamic permeability with the growth of coupling parameter in both cases. The increase of coupling is equivalent to the increase of the coefficient of micro-rotation viscosity, i.e. micro-level effects; this leads to the slowing down the flow and, hence, the dropping down of the hydrodynamic permeability of the membrane. The influence of porous layer parameter $\sigma$ on the hydrodynamic permeability in Fig.2b demonstrates analogous tendencies as for the non-polar liquid investigated in the paper of Vasin (*Colloid J.*, 2010, Vol. 72, No. 3, 305–313). It is located obviously higher than analogous curves for polar liquid, because polar flow is usually slower than non-polar one. The scale parameter σ can be associated with the filtration characteristics of a porous medium.

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## References


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